Resonant generation and energetics of wind-forced near-inertial motions in a geostrophic flow

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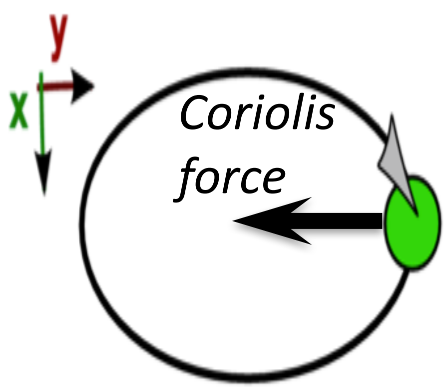
Inertial Oscillations

Inertial Frequency:

$$f = 2\Omega \sin(latitude)$$

$\Omega =$ angular frequency of Earth ~7.3E-5 rad/s^{-1}

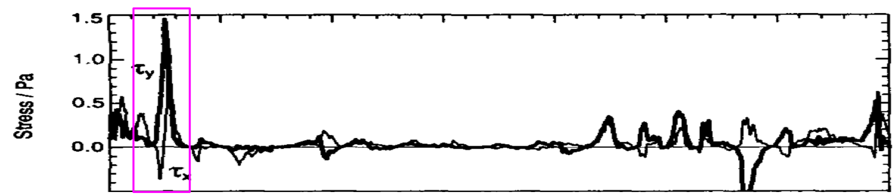
Force Diagram



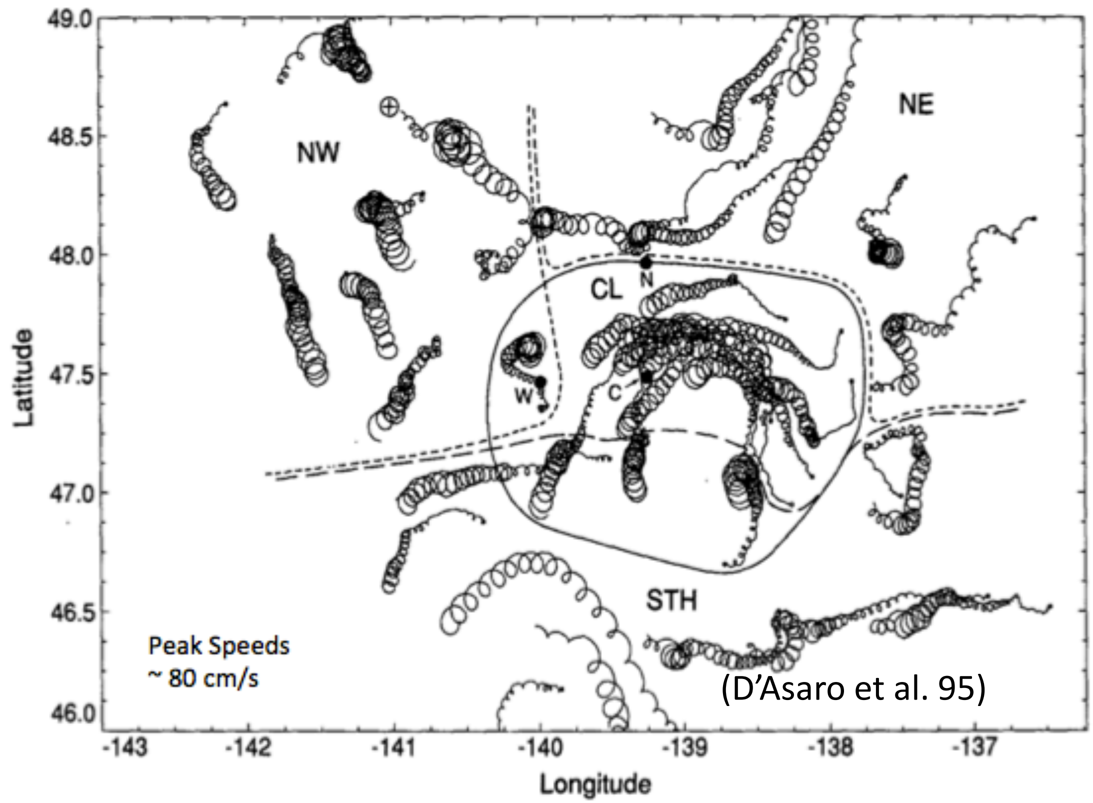
$$\frac{\partial u}{\partial t} - fv = 0$$

$$\frac{\partial v}{\partial t} + fu = 0$$

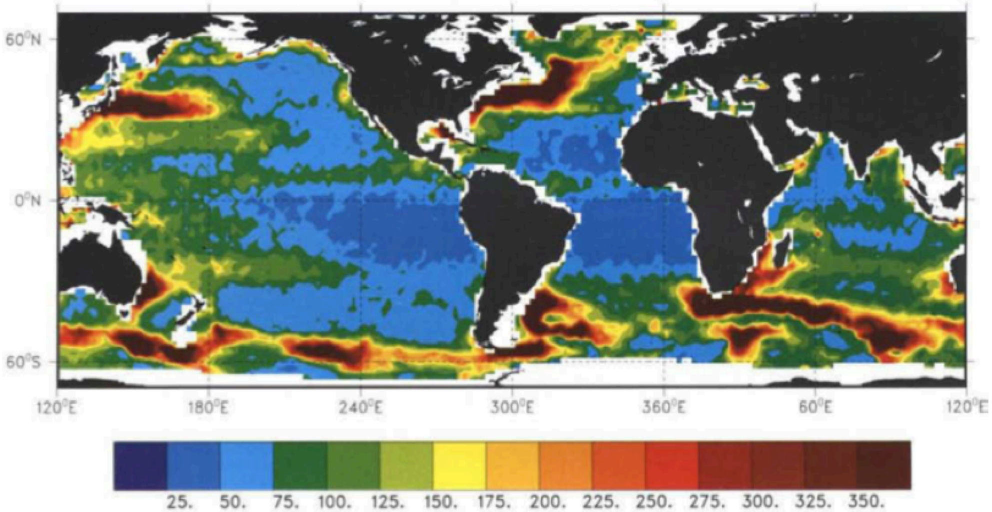
35 days of surface wind stress



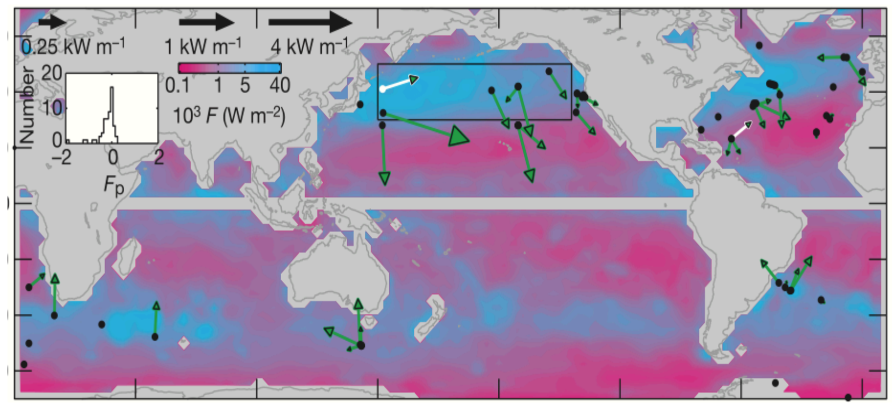
25 days of drifter trajectories



Mesoscale eddy kinetic energy (EKE) [Wunsch 2002]



Energy flux from winds to mixed-layer near-inertial motions [Alford 2003]

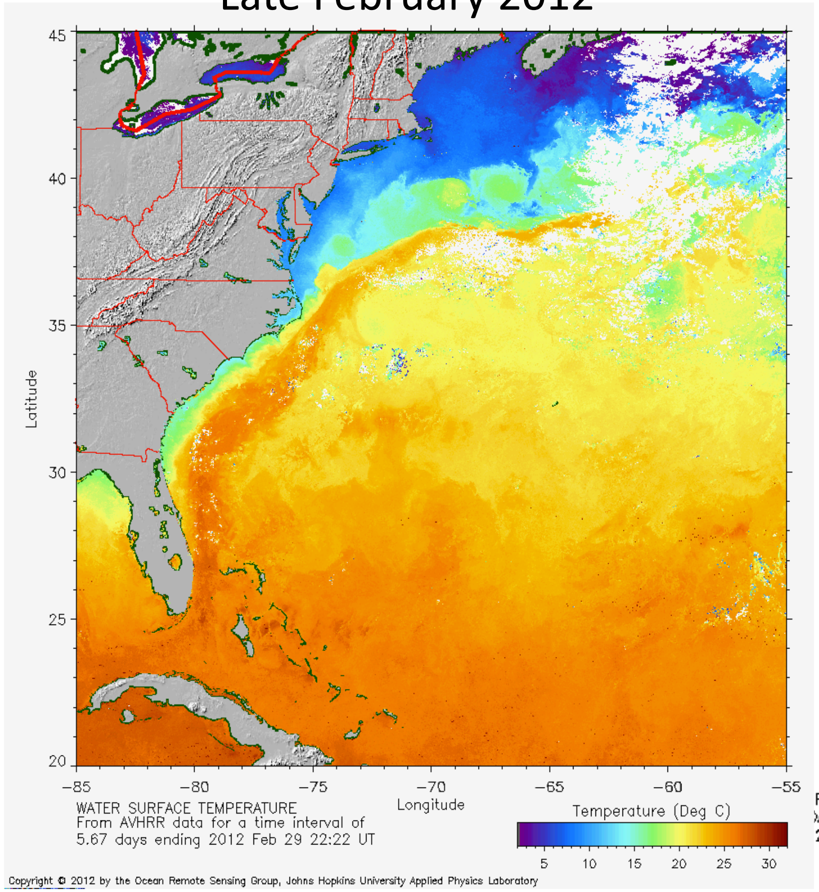


Blue is larger/pink is smaller!

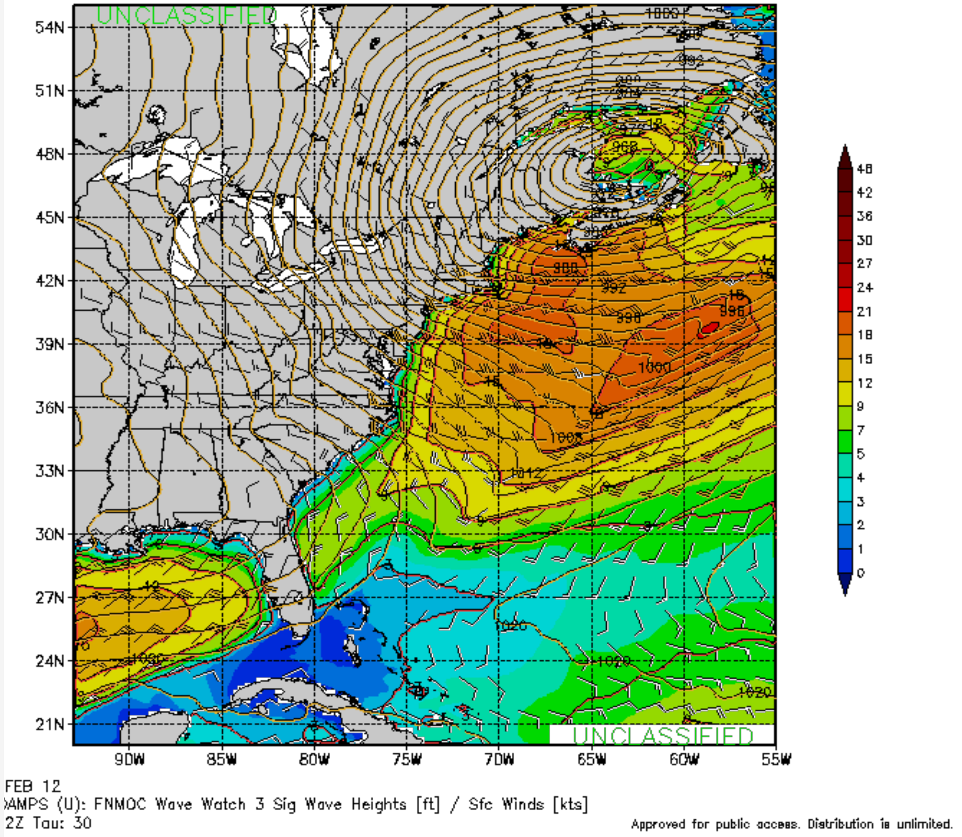
- ~1 TW flux into inertial oscillations.
- Much of the energy enters in regions of high EKE

Atmospheric mesoscale ~ 100-1000 km, Oceanic mesoscale ~ 10-100 km

Sea Surface Temperature,
Late February 2012



Surface Weather Chart,
Late February 2012

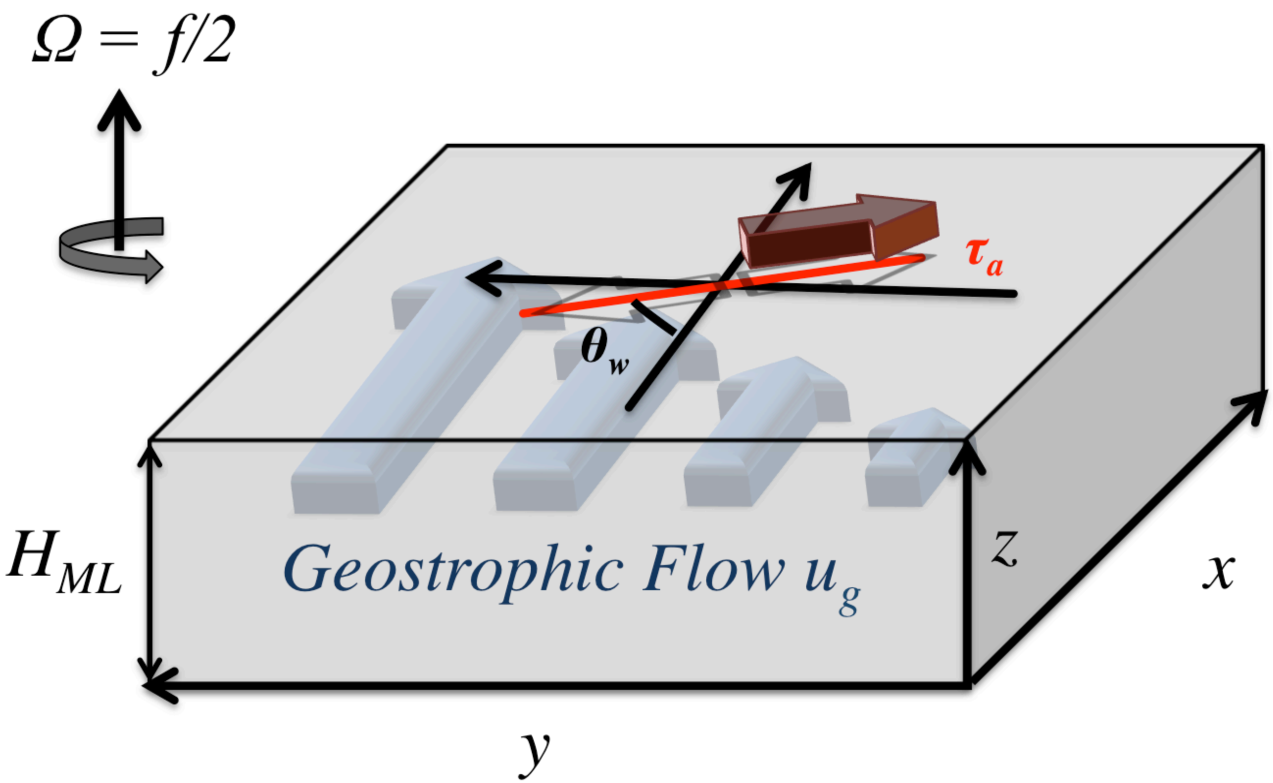


Scale Separation

How does an axisymmetric jet $u_g(y)$ modify <u>generation</u> of near-inertial motions by spatially-uniform oscillatory winds?

Wind stress oscillates along a line

$\omega_{wind} = constant$



What is an appropriate model when *du/dy ~ f*?

How does wave amplitude depend on: du/dy, θ, ω?

Dimensionless Perturbation Momentum Eqns.

Scale separation -> model reduction

$$\frac{\partial u'}{\partial t'} + \mathrm{Ro}_a\left(v'\frac{\partial u'}{\partial y'} + w'\frac{\partial u'}{\partial z'}\right) - v'\left(1 - \mathrm{Ro}_g\frac{\partial u'_g}{\partial y'}\right)$$

$$+ \mathrm{Fr}_g\mathrm{Bu}_a^{1/2}w'\frac{\partial u'_g}{\partial z'} = \mathrm{Ek}_a X',$$

$$\frac{\partial v'}{\partial t'} + \mathrm{Ro}_a\left(v'\frac{\partial v'}{\partial y'} + w'\frac{\partial v'}{\partial z'}\right) + u' + \mathrm{Bu}_a\frac{\partial p'}{\partial y'} = \mathrm{Ek}_a Y',$$

Wave scales << 1

$$\mathrm{Bu}_a = \left(\frac{\tilde{N}\tilde{H}_a}{f\tilde{L}_a}\right)^2 \quad \mathrm{Ro}_a = \frac{\tilde{U}_a}{f\tilde{L}_a}$$

Larger geostrophic scale

$$\bar{\mathrm{Ro}}_g = \tilde{U}_g/f\tilde{L}_g \sim 1$$

Modeling the mixed-layer-average momentum in a jet: a local slab model

Time Dependence

Parameterized Radiative Decay/Local Dissipation

Wind Stress/Body Force

$$\frac{\partial U_{ML}}{\partial t} - \frac{F^2}{f} V_{ML} = -rU_{ML} + \frac{\tau_x}{\rho H_{ML}}$$

$$\frac{\partial V_{ML}}{\partial t} + fU_{ML} = -rV_{ML} + \frac{\tau_y}{\rho H_{ML}}$$

U_{ML}, V_{ML}, τ_x, τ_y function of time only

$$F^2 = f(f - \partial u_g/\partial y) = f^2(1 + Ro_g)$$

(see also Pollard & Millard 1970, Stern 1975, Weller 1982, Klein & Hua 1988, Klein & Treguier 93)

Classic Driven Harmonic Oscillator

$$\frac{\partial^2 U_{ML}}{\partial t^2} + 2r\frac{\partial U_{ML}}{\partial t} + \left(r^2 + F^2\right) U_{ML} = \frac{1}{\rho_0 H_{ML}}\left(\frac{\partial \tau_x}{\partial t} + r\tau_x + \frac{F^2}{f}\tau_y\right)$$

*This system is under-damped and susceptible to **resonance.***

General Solution

$$U_{ML}(t) = U^H(t) + U^P(t)$$

Unforced Part

$$U_{ML}^H = D_1 e^{-rt}\cos(Ft) + D_2 e^{-rt}\sin(Ft)$$

Forced Part

$$U_{ML}^p(t) = A_U \sin(\omega_w t + \phi_{U^P})$$

Inviscid Initial Value Problem

Resonant frequency $F = \sqrt{f\left(f - \frac{\partial u_g}{\partial y}\right)}$

Dynamics

$$\frac{\partial U_{ML}}{\partial t} - \frac{F^2}{f} V_{ML} = 0,$$
$$\frac{\partial V_{ML}}{\partial t} + f U_{ML} = 0.$$

$$\frac{\partial^2 \eta}{\partial t^2} + F^2 \eta = 0$$

$$\eta(T) = \int_0^T v\, dt$$

Equivalent Lagrangian Description

Energetics

$$E_{ML}(t) = \frac{U_{ML}(t)^2 + V_{ML}(t)^2}{2}$$

$$= E_{ML}(0) + \underbrace{\int_0^t -\frac{\partial u_g}{\partial y} V_{ML}(s) U_{ML}(s)\, ds}_{\text{LSP}}$$

Lateral Shear Production

Constant wave activity A ≠ E

$$A = v^2 + \frac{u^2}{1 + Ro_g} = \eta_t^2 + F^2 \eta^2$$

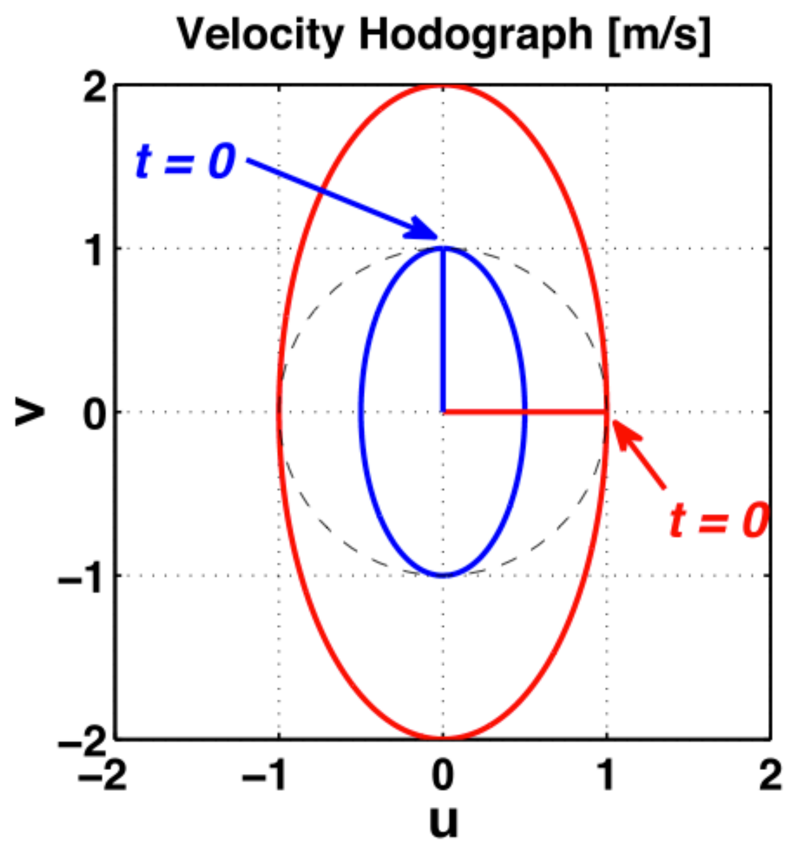
$$\left\langle v^2 \right\rangle = F^2 \left\langle \eta^2 \right\rangle$$ Phase/ensemble avg.

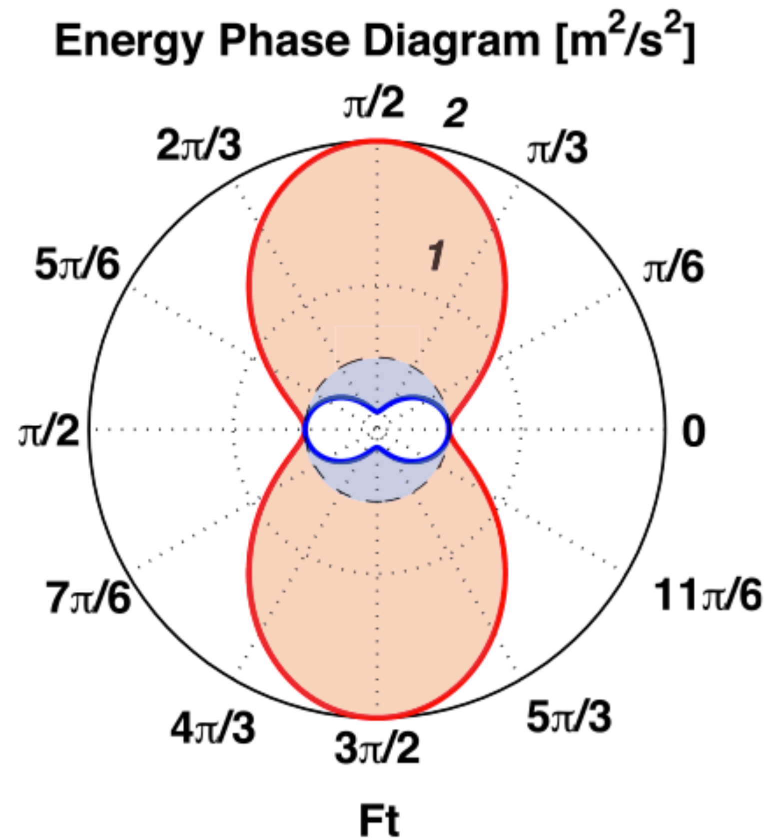
Wave activity equipartition

Example Inviscid Initial Value Problem

Resonant frequency

$$F = \sqrt{f\left(f - \frac{\partial u_g}{\partial y}\right)} = .5f$$



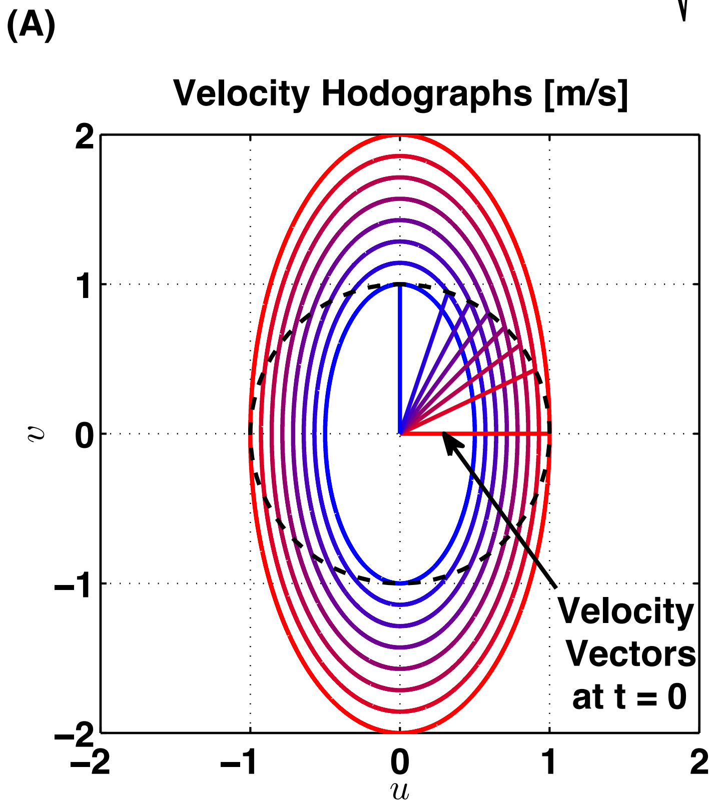


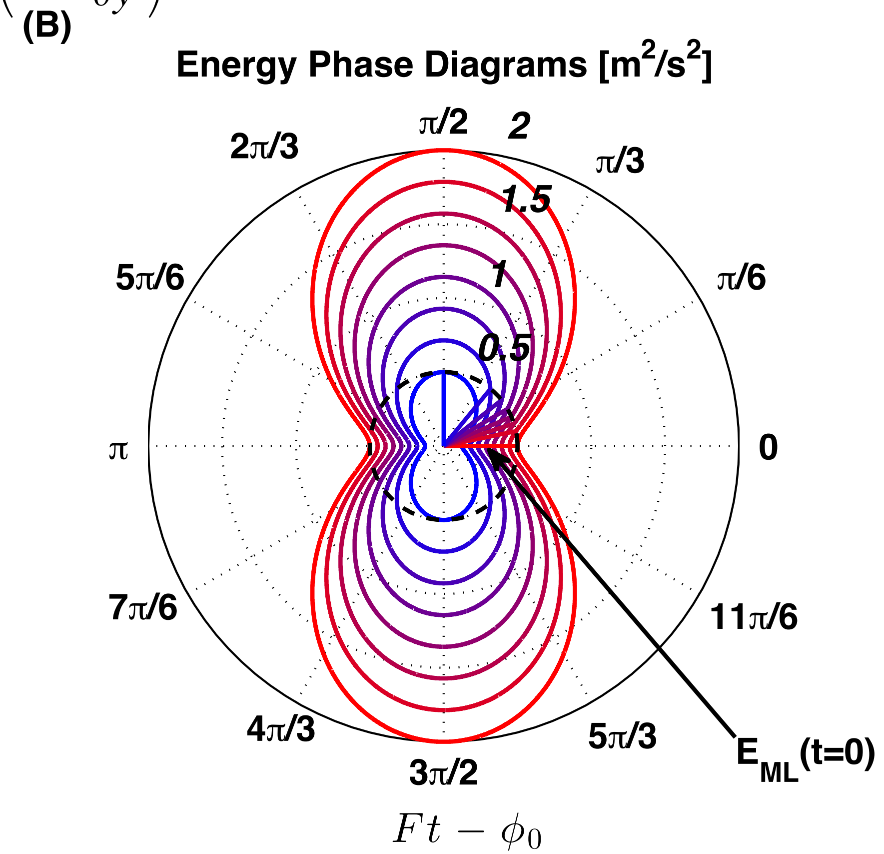
- Waves exchange energy with the background flow
- Velocity hodographs are elliptic.
- Wave energy returns to its initial value at $Ft = 2\pi$.

Ensemble of Inviscid Initial Value Problems

Resonant frequency

$$F = \sqrt{f\left(f - \frac{\partial u_g}{\partial y}\right)} = .5f$$





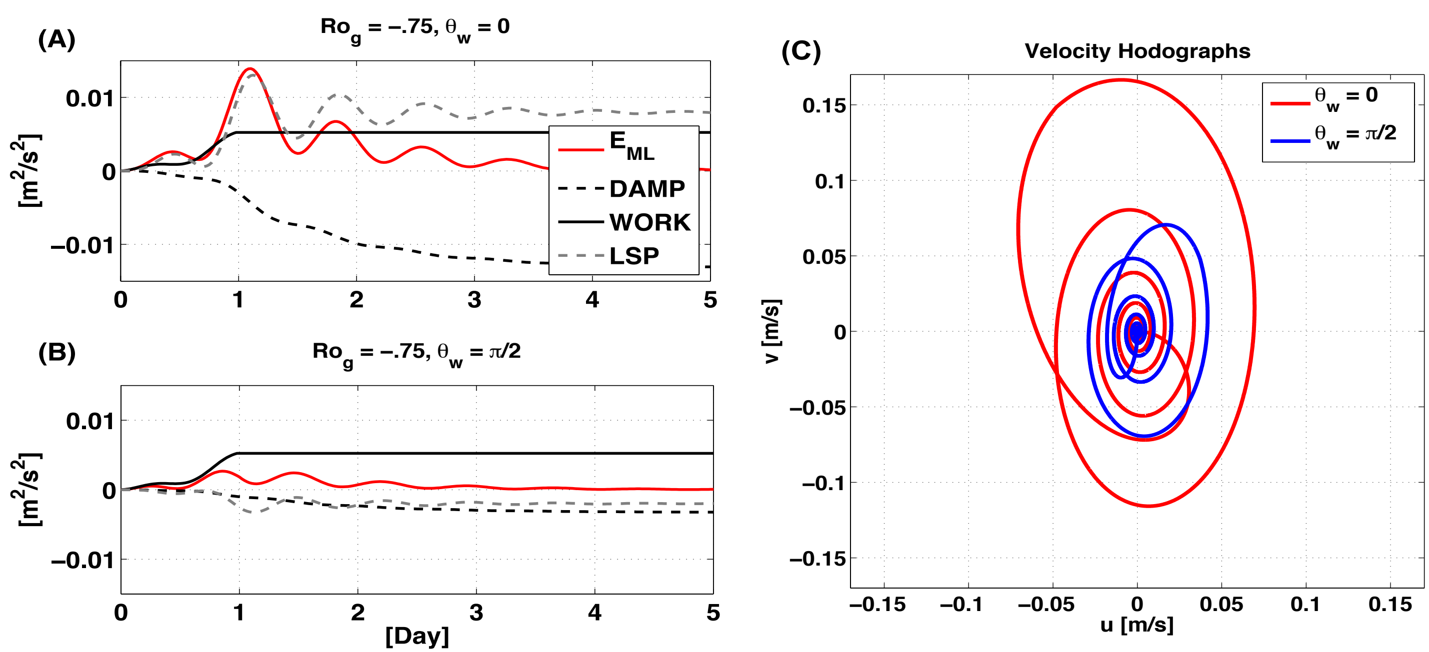
- Time and ensemble-averaged wave energy is greater than the initial energy.

Transient Forced and Damped Problem

- Forcing + dissipation drive time-integrated energy exchange between waves and geostrophic flow via LSP.

- Sign + magnitude of energy exchange depend on wind direction and geostrophic vorticity.

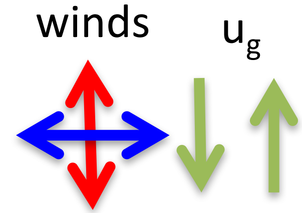
$$E_{ML}(t) - E_{ML}(0) = \underbrace{\int_0^t -\frac{\partial u_g}{\partial y} V_{ML}(s) U_{ML}(s)\, ds}_{\text{LSP}} + \underbrace{\int_0^t -2r E_{ML}(s)\, ds}_{\text{DAMP}} + \underbrace{\int_0^t \frac{\boldsymbol{\tau}(s)\cdot \mathbf{U}_{\mathbf{ML}}(s)}{\rho_0 H_{ML}}\, ds}_{\text{WORK}}$$



Resonant frequency

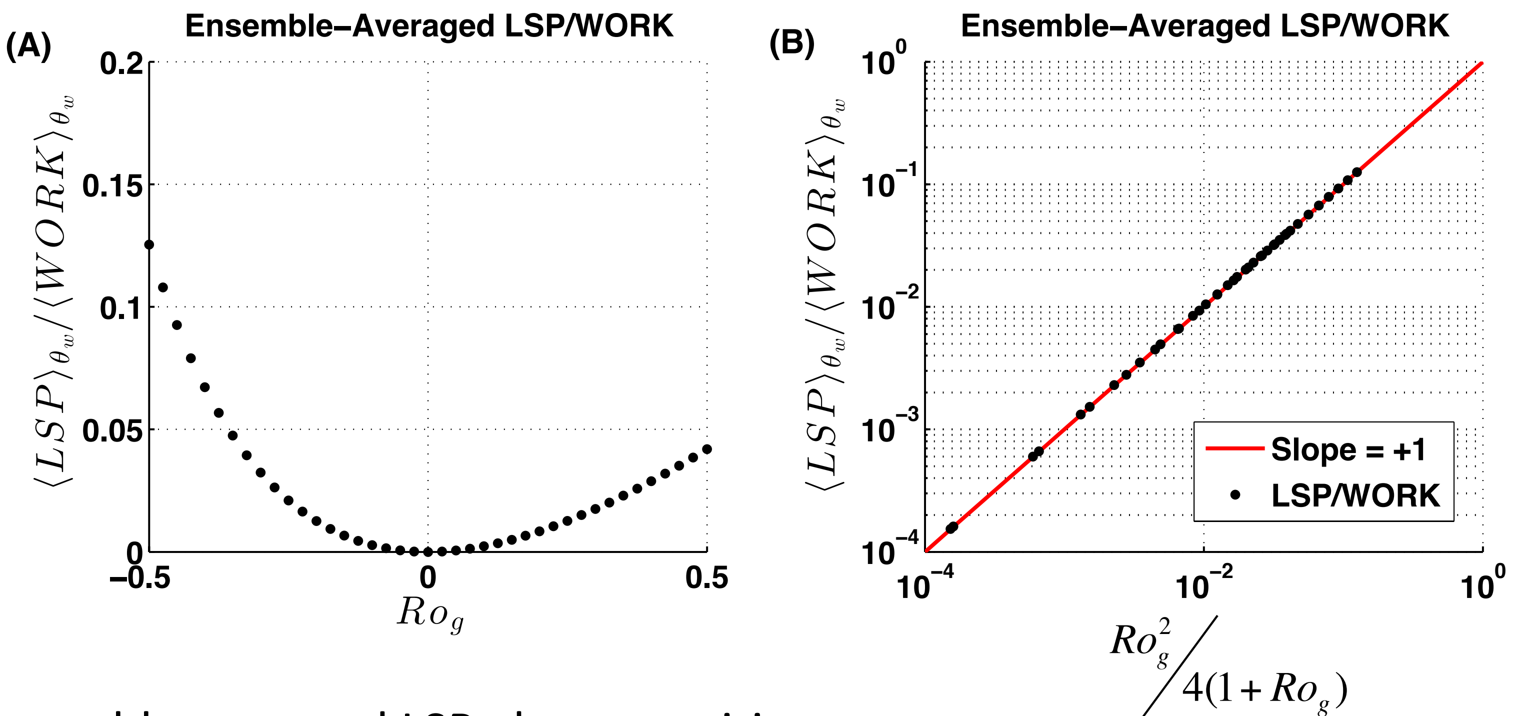
$$F = \sqrt{f\left(f - \frac{\partial u_g}{\partial y}\right)} = .5f$$

$\boldsymbol{\omega_{wind}}$ **=F**



Ensemble of Transient Forced Problems

forced by winds with an isotropic distribution of angles

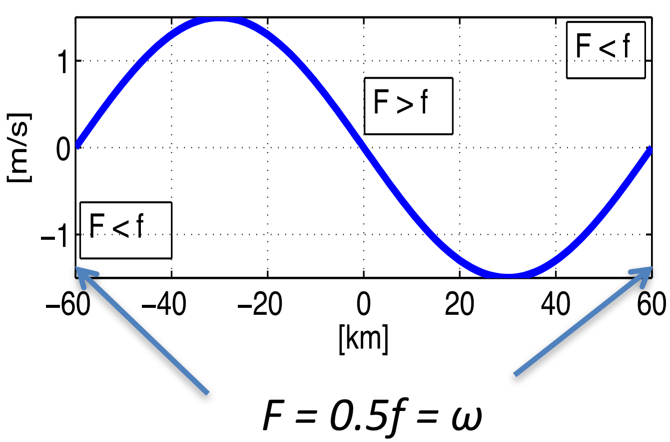


- Ensemble averaged LSP always positive
  => net energy transfer from mean to an ensemble of waves.
- Exchange is larger for anti-cyclonic vorticity
  => will tend to skew vorticity distribution of geostrophic flow toward positive values

$$\langle LSP\rangle_{\theta} \sim \langle WORK\rangle_{\theta} \frac{Ro_g^2}{4(1+Ro_g)}$$

Some Numerical Simulations
Hydrostatic Boussinesq Equations

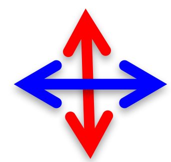
Jet Velocity

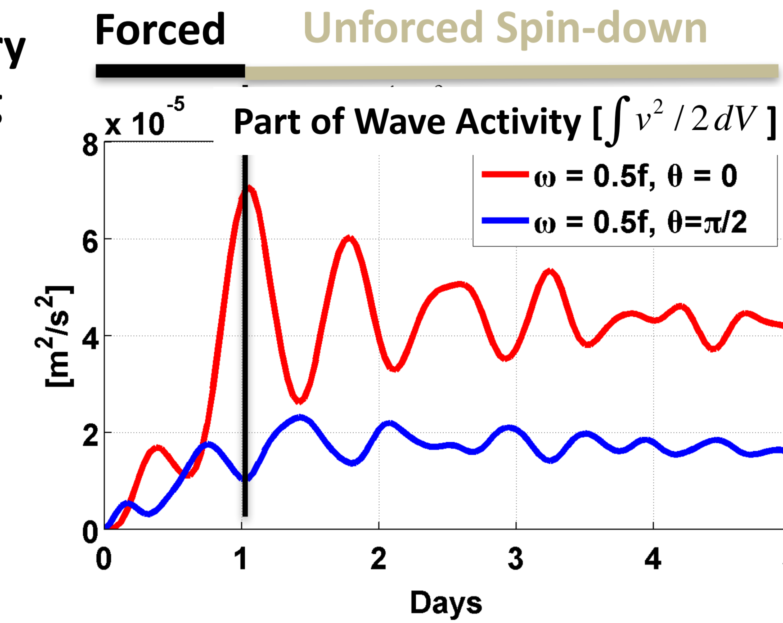


24 h Oscillatory Wind Forcing

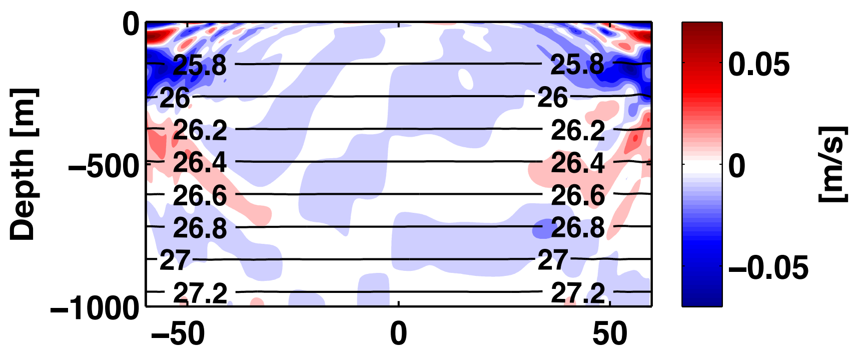
θ=0, θ=π/2

ω=0.5f

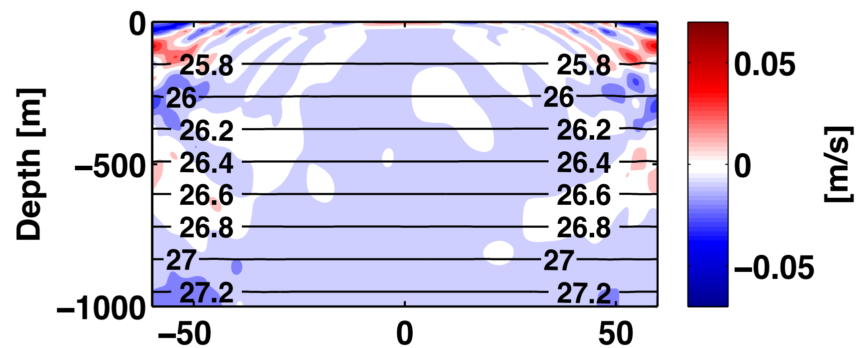




Cross-stream velocity, θ=0, @ 5 days



Cross-stream velocity, θ=π/2, @ 5 days



$$f = 1\times10^{-4}\,s^{-1}, N^2 = 2\times10^{-5}\,s^{-2}$$

Linearized wave activity is conserved in the absence of transience/diabatic/viscous effects

Constant time-dependent wave activity

Initial wave activity

$$\int A dV = \int_V \eta_t^2 + F^2\eta^2 + N^2\zeta^2 dV = \int_V v_0^2 + \frac{u_0^2}{1+Ro_g} dV$$

$$\eta(T) = \int_0^T v\,dt, \zeta(T) = \int_0^T w\,dt$$

$$\left\langle v^2 \right\rangle = \left\langle F^2\eta^2 + N^2\zeta^2 \right\rangle$$

Wave activity equipartition in the ensemble/phase average.

Easily generalizes for a baroclinic flow

(see Whitt and Thomas 2013 JPO)

Some Broader Implications

- Near-inertial waves forced by isotropic winds dissipate ***$O(Ro_g^2)$*** fraction of energy from the mean flow at small Ro.
- At **O(1) Ro**
  - KE input from winds to unbalanced motions may catalyze extraction of KE from balanced flow in proportion to wind input
  - Balanced -> unbalanced KE transfer is greater in anti-cyclonic vorticity -> may contribute to cyclone/anti-cyclone asymmetry in upper ocean