Off-Equatorial Deep-Cycle Turbulence Forced by Tropical Instability Waves in the
Equatorial Pacific

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ABSTRACT: The equatorial Pacific cold tongue is a site of large heat absorption by the ocean. This heat uptake is enhanced by a daily cycle of shear turbulence beneath the mixed layer—“deep-cycle turbulence”—that removes heat from the sea surface and deposits it in the upper flank of the Equatorial Undercurrent. Deep-cycle turbulence results when turbulence is triggered daily in sheared and stratified flow that is marginally stable (gradient Richardson number $Ri \approx 0.25$). Deep-cycle turbulence has been observed on numerous occasions in the cold tongue at $0^\circ$, $140^\circ$W, and may be modulated by tropical instability waves (TIWs). Here we use a primitive equation regional simulation of the cold tongue to show that deep-cycle turbulence may also occur off the equator within TIW cold cusps where the flow is marginally stable. In the cold cusp, preexisting equatorial zonal shear $u_c$ is enhanced by horizontal vortex stretching near the equator, and subsequently modified by horizontal vortex tilting terms to generate meridional shear $v_c$ off of the equator. Parameterized turbulence in the sheared flow of the cold cusp is triggered daily by the descent of the surface mixing layer associated with the weakening of the stabilizing surface buoyancy flux in the afternoon. Observational evidence for off-equatorial deep-cycle turbulence is restricted to a few CTD casts, which, when combined with shear from shipboard ADCP data, suggest the presence of marginally stable flow in TIW cold cusps. This study motivates further observational campaigns to characterize the modulation of deep-cycle turbulence by TIWs both on and off the equator.

KEYWORDS: Pacific Ocean; Tropics; Mixing; Turbulence

1. Introduction

The equatorial Pacific cold tongue is a zonal band of cold upper-ocean water in the eastern equatorial Pacific extending between the Pacific’s eastern margin and approximately 155°W. Being cold water in a region of large solar insolation, the cold tongue is a region of large heat absorption by the ocean between approximately 3°S and 3°N (e.g., Large and Yeager 2009; Holmes et al. 2019). Absorbing such a large amount of heat while maintaining cold sea surface temperatures (SSTs) is only possible if the heat taken in by the ocean is quickly transported away from the equatorial sea surface. Mixed layer heat budgets for the eastern Pacific constructed using equatorial mooring observations show that the downward transport of heat by microscale turbulence is essential to keeping the sea surface cool (Moum et al. 2013; Wang and McPhaden 1999, 2000; Warner and Moum 2019; Ray et al. 2018). The annual cycle of turbulence is strong enough that there is only a single peak in the annual cycle of equatorial sea surface temperature (SST) despite a double peak in solar insolation as the sun “crosses” the equator twice through the year (Moum et al. 2013). On longer time scales, the rate of change of SST during ENSO phase transitions approximately agrees with both amplitude and phase of turbulence heat flux divergence across the mixed layer (Warner and Moum 2019). Equatorial mixing might be critical to one of Earth’s most consequential climate phenomena. Yet little is known about its spatial structure with current observations restricted to a few sites along the equator.

The microstructure observations that do exist have illustrated the complex nature of equatorial turbulence. At the equator, winds maintain a strong zonal mean flow with intense vertical shear between the eastward flowing Equatorial Undercurrent (EUC) at depth and the westward flowing near-surface South Equatorial Current (SEC). Observations at $0^\circ$, $140^\circ$W repeatedly show the existence of diurnally modulated shear-driven turbulence beneath the base of the mixed layer in the high shear zone above the core of the EUC—“deep-cycle turbulence” (Gregg et al. 1985; Moum and Caldwell 1985; Lien et al. 1995; Moum et al. 2009). In this deep-cycle layer the flow’s gradient Richardson number,

$$Ri = \frac{g}{\rho_0} \frac{\partial u}{\partial z} + \frac{\partial v}{\partial z},$$

(1)

(where $b = -g\rho_0$ is buoyancy, $u$ and $v$ are the horizontal components of velocity, and subscripts indicate differentiation), remains near its critical value of 0.25 throughout the year. That is, this layer is marginally stable to shear instability (Thorpe and Liu 2009; Smyth and Moum 2013; Pham et al. 2017; Smyth et al. 2019).
A daily cycle of turbulence requires a diurnal trigger in the marginally stable layer. At least two hypotheses have been proposed for this diurnal trigger. A number of authors (e.g., Wijesekera and Dillon 1991; Peters et al. 1994) propose that downward-propagating internal waves, excited at the base of the mixed layer during nighttime convection, break in the marginally stable layer and trigger turbulence. However, the 1D model solutions of Schudlich and Price (1992) suggested that the diurnal trigger is the descent of the daytime shear layer formed by the trapping of momentum in a near-surface stratified layer. Their model result supported observations reported in Smyth and Moum (2013).

As the near-surface stratification built up by solar radiation during the day decreases in the afternoon the warm-layer jet goes unstable, triggering turbulence at the base of the mixed layer in large-eddy simulations (Pham et al. 2017). This turbulence transports momentum downward, locally increasing shear in the marginally stable layer, reducing Ri and triggering further shear instability. As this process continues, the envelope of actively turbulent fluid propagates downward and is visible as a downward descent of a local maximum in vertical shear (Smyth et al. 2013). Turbulence persists during the next morning at depths up to 80–100 m at 0°, 140°W despite the cessation of near-surface convection; and increases Ri back to its marginally stable value of approximately 0.25 at which point turbulence decays. Smyth et al. (2019) argue that turbulent diffusion tends to reduce buoyancy and velocity gradients, $b_z$ and $u_z$, at roughly similar rates so $\text{Ri} = b_z/u_z$ must increase. In this way flow in the deep-cycle layer is restored to a state of marginal stability ($\text{Ri} \approx 0.25$). Near-surface turbulence is then triggered in the late afternoon and the cycle repeats. Large-eddy simulations initialized with mean fields representative of each of the four seasons show that deep-cycle turbulence exists throughout the year, though the depth to which it penetrates varies with the seasonal shoaling and deepening of the EUC (Pham et al. 2017). Mooring observations in the Atlantic Ocean (Wenegrat and McPhaden 2015), ship-based microstructure measurements in the Indian Ocean (Pujiana et al. 2006) and Holmes and Thomas (2015), and a global primitive equation simulation using a $k$–$e$ turbulence closure model (Pei et al. 2020) suggest that deep-cycle turbulence may also exist in the equatorial Atlantic and Indian Oceans. These studies have all focused on deep-cycle turbulence right at the equator.

The equatorial Pacific is also the site of energetic mesoscale variability in the form of tropical instability waves (TIWs). TIWs are formed through barotropic and baroclinic instability of the equatorial current system, and manifest as a series of westward traveling cold cusps and warm troughs. TIWs have zonal wavelengths of 1000–2000 km, periods of 15–40 days, and propagate westward with a phase speed of approximately 0.5 m s$^{-1}$ (Philander 1976; Legeckis 1977; Qiao and Weisberg 1995; Chelton et al. 2000; Lyman et al. 2005; Holmes and Thomas 2016). The cold cusps of TIWs exhibit vigorous circulation features with sharp fronts and convergence zones that influence biological activity (Yoder et al. 1994; Strutton et al. 2001; Warner et al. 2018).

TIWs may modulate cold tongue turbulence. Lien et al. (2008) reported intense mixing at the base of the mixed layer recorded by a Lagrangian float that meandered through two TIWs between 2°S and 1°N in 2005 with a peak heat flux estimate of 1000 W m$^{-2}$. During the EQUIX experiment at 0°, 140°W in 2008, a La Niña year, turbulent heat fluxes at the equator reached 400 W m$^{-2}$ when averaged over 2 weeks (Moum et al. 2009) in the presence of TIWs, compared to the approximately 50 W m$^{-2}$ measured previously (Lien et al. 1995). The base of the marginally stable layer appears to shallow by a factor of 2 to 50 m during the warm phase of the TIW as compared to that during the cold phase (Moum et al. 2009; Inoue et al. 2012, 2019). Recent work with TAO observations indicates that a subsurface mode of TIWs (Luu et al. 2019) as well as equatorial inertial gravity waves at periods between 3 and 25 days (Liu et al. 2020) may strongly influence the vertical structure of shear and thereby diapycnal mixing at the equator, consistent with (Moum et al. 2009; Inoue et al. 2019). Note that microstructure profiler observations have not yet captured one entire TIW period. These observations are consistent with modeling studies that show enhanced turbulence both at the equator during particular TIW phases (Menkes et al. 2006; Holmes and Thomas 2015). Holmes and Thomas (2015) proposed a dynamical explanation for the enhancement of equatorial turbulence by TIWs: meridional diffusion (positive $v_z$) of TIW flow at the equator during the cold phase stretches the southward-oriented meridional component of horizontal vorticity ($\omega^r = w_x - u_y$). The stretching increases $|\omega^r|$, intensifies the preexisting EUC shear $u_y$ at the equator, and forces intense turbulence. During the warm phase, the reverse happens: vortex tubes are squashed and $u_y$ weakens with a corresponding weakening of turbulence. Observational evidence for this mechanism is inconclusive, due in part to the difficulty in obtaining an observational estimate of $v_z$ (Inoue et al. 2019). TIWs also change the latitudinal scale of equatorial mixing: the simulations of both Menkes et al. (2006) and Holmes and Thomas (2015) show the presence of intense mixing in TIW cold cusps off the equator. But, most equatorial microstructure measurements have been made right at the equator, primarily at 0°, 140°W in the Pacific. This sampling bias means that large unknowns remain in our knowledge of the spatial distribution of equatorial upper-ocean mixing.

Here we use a 1/20° regional model of the cold tongue to study how TIWs modulate turbulence off the equator. The simulated TIWs force off-equatorial marginally stable flow and deep-cycle turbulence by generating intense meridional shear ($v_z$) in the eastward extension of a TIW’s cold cusp (sections 3 and 4). This shear results from the rotation or tilting of horizontal vorticity generated by horizontal vortex stretching at the equator (section 4b). Indirect observational evidence for this TIW-forced off-equatorial deep-cycle turbulence is presented through Ri profiles from three cruise sections through TIWs at 110°W (section 5). Our results emphasize the need to observe turbulence variability off the equator; to validate the results presented here, to assess the accuracy of turbulence parameterization schemes in simulating TIW modulated deep-cycle turbulence, and to fully characterize the role of deep-cycle turbulence in the heat budget of the cold tongue.
2. Methods

a. Model configuration

We use a regional configuration of the MITgcm (Marshall et al. 1997; Adcroft et al. 2004) to model the equatorial Pacific cold tongue using NCAR’s Cheyenne cluster (Computational And Information Systems Laboratory 2019). The domain extends from 170°W to 95°W, 12°S to 12°N with a horizontal grid spacing of 1/20°. There are 345 vertical levels with a spacing of 1 m in the top 250 m that increases to a maximum of 250 m. The model is forced at the surface with fields from the JRA-55do reanalysis (Tsujino et al. 2018) and lateral boundary conditions specified using daily averaged fields from the Mercator GLORYS12V1 1/12° ocean reanalysis product (Copernicus identifier GLOBAL_REANALYSIS_PHY_001_030). The simulated time period is from 1 September 1995 to 28 February 1997. There is no tidal forcing. The time axis, when presented in figures, is local time at 110°W, chosen to be UTC − 7.

Subgrid-scale vertical mixing is parameterized using the K-profile parameterization (KPP) scheme of Large et al. (1994) with standard parameter values (Large and Gent 1999). The KPP turbulence scheme divides the water column into three parts: an interior region, a boundary layer, and a surface layer (defined to be the top 10% of the boundary layer near the surface). The KPP boundary layer depth $H_{KPP}$ is usually chosen as the depth at which the bulk Richardson number

$$\frac{\text{Ri}_b}{\text{Ri}_b} = \frac{bH_{0.3}}{\sqrt{|V|^2 + V_t|^2(H_{0.3})}}$$

exceeds a critical value chosen to be 0.3 ($H_{0.3}$). Here $b$ and $V$ are the differences between values of resolved buoyancy $b$ and resolved velocity $V$ at the surface and at the base of the boundary layer $z = H_{0.3}$. $V_t$ represents a parameterized velocity due to unresolved turbulent eddies. In addition, $H_{KPP}$ is restricted to be less that the Monin–Obukhov length scale

$$L_{MO} = \frac{-u_t^*}{\kappa B_0},$$

so that

$$H_{KPP} = \min[L_{MO}, H_{0.3}].$$

Here $u_t^* = \sqrt{\tau |b_0|}$ is the friction velocity where $\tau$ is the wind stress, $B_0$ is the surface buoyancy flux, and $\kappa = 0.4$ is the von Kármán constant. Below $H_{KPP}$, diffusion due to shear instabilities is parameterized using a diffusivity that is a smooth function of gradient Richardson number $\text{Ri}$. This function is nonzero for $\text{Ri} < 0.7$ (Large et al. 1994, their Fig. 3). The shear mixing scheme’s parameters were specifically tuned to reproduce the onset of nighttime convection relative to LES (Large and Gent 1999).

b. Model validation

The model simulates the equatorial Pacific reasonably well (Fig. 1). The model fields are compared with the Johnson et al. (2002) climatology and an annual mean climatology constructed using TAO mooring observations in the 1990–2000 decade at 110°W. At the equator, the EUC maximum is approximately 15–20 m deeper than the TAO data and the Johnson et al. (2002) climatology (Fig. 1b), though the depth-integrated zonal velocity in the top 250 m is comparable to that from the Johnson climatology (Fig. 1c). There is a subsurface warm bias relative to both observational datasets (Figs. 1a,e). This subsurface warm bias extends from about 6°S and 6°N, where the top 75 m are slightly stratified in temperature than observed.

c. Diagnostics

All derivatives in presented terms are estimated using centered differences. Ri is masked out when $N^2 < 10^{-6}$ s$^{-2}$ or $S^2 < 10^{-6}$ s$^{-2}$. The mixed layer depth $z_{MLD}$ is computed as the shallowest depth at which the density exceeds the surface density by 0.015 kg m$^{-3}$. The base of the deep-cycle layer $z_{BI}$ is computed as the shallowest depth below the mixed layer base when $\text{Ri}$ exceeds 0.5. Our choice of 0.5 instead of 0.25 is discussed in section 3. The thickness of the low Richardson number layer or “low $\text{Ri}$ layer” is defined as $H = z_{MLD} - z_{RI}$. The parameterized turbulent heat flux $J'_q = -\rho_cp_R\frac{\theta}{\theta_T}(T - \gamma_T)$, where $c_p$ is the specific heat capacity of water, and $\gamma_T$ is a nonlocal transport that is nonzero only during convective forcing conditions [Large et al. 1994, their Eqs. (19), (20)]. Negative values of $J'_q$ indicate heat moving downward.

3. TIW modulated turbulence in the cold tongue

TIW variability imprints itself strongly on SST, surface heat fluxes, and subsurface turbulent mixing both on and off the equator (Fig. 2). The cold tongue, structured by TIW cold cusps, is characterized by strong heat absorption at the surface (positive $Q_{net}$ in Fig. 2a) and intense downward turbulent heat fluxes in the thermocline (Fig. 2d). The low $\text{Ri}$ layer also coincides with the cold tongue suggesting both TIW influence and the existence of intense vertical mixing (Fig. 2e). In contrast, the EUC does not coincide with the cold tongue; it is concentrated in an approximately 1° wide latitudinal band around the equator, particularly to the east of 125°W (Fig. 2c). The off-equatorial low $\text{Ri}$ flow (Fig. 2e) and associated turbulent heat fluxes (Fig. 2d) cannot be directly associated with shear in the EUC.

Though this manuscript subsequently focuses on one particular TIW to elucidate the processes driving the turbulent heat fluxes below the cold tongue, the correlation between low $\text{Ri}$ flow off the equator and cold SSTs associated with TIWs is visible throughout the simulation at 110°, 125°, 140°, and 155°W (Fig. 3). Low $\text{Ri}$ values are more common north of the equator than south of the equator, and the latitudinal extent of low $\text{Ri}$ flow widens toward the west. These patterns match those of SST and are seen outside the latitudinal extent of the EUC (black lines in Fig. 3). Later during the season (January–March) TIW amplitudes weaken, and clear cusps are not visible in SST. Even then low $\text{Ri}$ values are still visible off the equator where relatively cold SSTs are present (e.g., at 110°W in March; Figs. 3a,b) These patterns are qualitatively similar to those in Figs. 2d and 2e. The rest of this manuscript focuses on one representative TIW highlighted by the black box in Fig. 2d.
The vertical structure of mixing associated with this particular TIW at 110°W (black box in Fig. 2d) shows a deep cycle of mixing both on and off the equator at 3.5°N (Figs. 4c,e). At both latitudes, there is a daily cycle in the parameterized turbulent heat flux beneath the mixed layer base \((z_{MLD})\) orange line in Figs. 2c and 2e) and above the base of the low Ri layer \((z_{Ri})\); black line in Figs. 2c and 2e). At the equator, \(z_{Ri}\) shallows during the TIW warm phase as observed and described by Inoue et al. (2012, 2019) and Moum et al. (2009). At 3.5°N, deep-cycle turbulence starts when the cold cusp reaches that latitude (around 26 November), and persist over the approximately 2-week time period during which the TIW cold cusp reaches this latitude. There is almost no turbulence beneath the mixed layer prior to the arrival of the cold cusp and after its
departure. Daily cycles are also visible in Ri, $S^2$, $N^2$ at 3.5°N beneath the mixed layer base for the time period when deep-cycle turbulence is active (Fig. 5). Such daily cycles in shear, stratification and Ri are qualitatively similar to observations and LES at 0°, 140°W (Moum et al. 2009; Pham et al. 2017).

A robust feature of observed deep-cycle layers at the equator is a median Ri of 0.25 (Smyth and Moum 2013; Pham et al. 2017). In the simulated deep-cycle layer both on and off the equator, Ri is indeed low, similar to observations, but the median value is higher at about 0.4, a notable bias (Figs. 4d,f), suggesting that the KPP mixing scheme is too diffusive. This may be partly responsible for the subsurface warm bias in Fig. 1e. The critical Ri bias may partly arise because KPP’s shear mixing scheme parameterizes diffusivity $K_T$ as a smooth function of Ri that is nonzero for $Ri > 0.7$ (Large et al. 1994). This function lacks a rapid transition in diffusivity as Ri reduces, thereby making it difficult to represent a state of marginal instability at $Ri \approx 0.25$ (Holmes and Thomas 2015). In contrast, plots of diffusivity versus Ri from observations show a steeper dependence (e.g., Zaron and Moum 2009; Peters et al. 1988). We also find that approximately a third of the parameterized turbulent heat flux in the deep-cycle layer is handled by KPP’s surface layer mixing scheme (discussed later in section 4d). Hence deficiencies in KPP’s shear mixing scheme may not be the sole reason for the bias in Ri. Here we account for the bias in Ri by treating simulated flows with $Ri > 0.4$ as being marginally stable.

4. Dynamics of off-equatorial deep-cycle turbulence

A deep cycle of turbulence can only exist in equilibrium in a stratified fluid given a continuous source of shear that acts to reduce Ri; a diurnal trigger that initiates turbulence daily
which may then increase \( R_i \) (Smyth et al. 2019); and a process that maintains stratification within the deep-cycle layer. Together these act to maintain the stratified deep-cycle layer in a state of marginal stability. At 0°, 140°W equatorial winds maintain intense zonal shear \( u_z \) between the SEC and the EUC and the diurnal trigger may be the nighttime instability of the surface-trapped diurnal warm-layer jet (Pham et al. 2017) or the breaking of downward propagating internal waves triggered by nighttime convection (Wijesekera and Dillon 1991). In contrast, we will see that the simulated marginally stable flow and deep-cycle turbulence at 3.5°N, 110°W is forced by TIW meridional shear \( u_z \) (sections 4a, 4b) and the diurnal trigger is the daily descent of the surface mixing layer associated with the weakening of the stabilizing surface buoyancy flux in the afternoon (section 4d). We begin

**FIG. 3.** Low \( R_i \) seen in the off-equatorial region at (a) 110°, (c) 125°, (e) 140°, and (g) 155°W between September 1995 and March 1996. (b),(d),(f),(h) Median \( R_i \) in the region \( z_{MLD} - 40 \leq z \leq z_{MLD} \) daily averaged SST illustrating TIW activity. Black lines on all panels show the EUC maximum and its latitudinal extent defined using the latitudes north and south of the core at which the eastward velocity drops by a factor of 2. The latitudes are determined using \( u \) at the depth of maximum eastward velocity.
by describing the shear and stratification structure in the simulated off-equatorial deep-cycle layer (section 4a).

a. Marginal stability, shear, and stratification in the cold cusp

The off-equatorial low Ri layer is closely associated with the TIW’s cold cusp (Figs. 6a,b, also Figs. 2b,d,e and 3). The contributions of each vector component of the shear as well as the stratification to Ri in the low Ri layer below the cold cusp are decomposed and illustrated by reduced shear $Sh_{red}^z = (\overline{u^2} + \overline{v^2}) - N^2/Ri_c \geq 0$ (Fig. 6c), where $Ri_c = 0.4$ instead of the usual 0.25 following the discussion in section 3. The flow in much of the cold cusp is near marginal stability, though the components of $Sh_{red}^z$ vary considerably in space and time.
Between 1°S and 2°N and behind the cold cusp, the vertical shear of the flow is largely associated with \( u_z \) (Fig. 6d). North of 2°N, the shear is largely associated with \( y_z \) (Fig. 6e). Combining the shear fields (Figs. 6d,e) with the map of low Ri layer thickness and \( S_{\text{red}}^2 \) (Figs. 6b,c) suggests that \( u_z \) drives the flow toward marginal instability south of 2°N while \( y_z \) drives the flow toward marginal instability north of 2°N, in the eastward extension of the cold cusp. Since \( S_{\text{red}}^2 \) is a linear combination of contributions from \( N^2, u_z^2 \) and \( y_z^2 \), we split \( S_{\text{red}}^2 \) into two terms to evaluate whether \( u_z \) or \( y_z \) is individually large enough to overcome half the stratification necessary to drive the flow toward marginal stability,

\[
S_{\text{red}}^2 = \left( \frac{u_z^2 - N^2}{2\text{Ri}} \right) + \left( \frac{y_z^2 - N^2}{2\text{Ri}} \right).
\]  

Snapshots of heat flux \( J_{\text{q}} \), reduced shear \( u_z^2 + y_z^2 - N^2/\text{Ri} \) and the two terms on the RHS of (5) computed using \( \text{Ri} = 0.4 \) are shown in Figs. 7a–l at three different times indicated by vertical dashed lines in Fig. 6. Enhanced turbulence is present in the low Ri layer between \( z_{\text{MLD}} \) (orange) and \( z_{\text{Ri}} \) (black). The regions of enhanced turbulence coincide with positive values of reduced shear. Zonal shear \( u_z \) is responsible for shear turbulence both at the equator (Figs. 7c,g,k) and in the northward-oriented cold cusp between 0° and 3°N (Figs. 7f,g). The corresponding \( v_z \) is weak (Figs. 7h,i). This pattern reverses in the eastward extension of the cold cusp: \( v_z \) is strong but \( u_z \) is weak between 2° and 4°N (Figs. 7k,l). These three cross sections confirm that \( u_z \) acts to force turbulence in the near-equatorial region (1°S–2°N) while \( v_z \) forces the off-equatorial deep-cycle turbulence, as inferred from Figs. 6c,d,e.

The stratification \( N^2 \) in the low Ri layer below the cold cusp varies by approximately a factor of 2 between the equator and the eastward extension of the cold cusp (Figs. 6f and 10a). This variation is a result of intense mixing seen in the cold cusp as we show later. Next we examine the dynamics underlying the shear field.

b. Shear forcing of off-equatorial deep-cycle turbulence

The enhanced off-equatorial \( v_z \) is a consequence of the rotation of the horizontal vorticity vector by the TIW’s
off-equatorial flow (Fig. 8). Although the flow is unsteady, water parcels follow a vortex-like circular path between 1° and 4°N in the cold cusp (Dutrieux et al. 2008; Holmes et al. 2014), illustrated here by streamlines (green) calculated using velocities relative to an approximate TIW westward translation speed of 0.5 m s⁻¹. The translation speed was determined from Hovmöller plots of SST and 0–60 m depth-averaged \( v \) at latitudes between 0° and 4°N. The circular path is visible in both latitude–time plane at 110°W and in latitude–longitude plane at 1 December 1995 (Figs. 8a,b). Near-equatorial disturbances propagate faster than off-equatorial disturbances (Kennan and Flament 2000) so the streamlines may be a poor representation of pathlines there. Streamlines pass through the near-equatorial region of high zonal shear, then move northward and eastward approximately parallel to the SST front (black contour; 23.8°C). During this transit the horizontal vorticity vector \( \omega^h \) rotates from pointing southward to pointing westward. This rotation suggests that horizontal vortex tilting transforms \( \omega^h \) to \( \omega^v \) during the parcel’s transit, i.e., negative \( u_z \) in the near-equatorial region is rotated to become positive \( v_z \) off the equator.

The dynamics of this transformation can be quantified using the evolution equation for the two shear components.

\[
D_{t} u_z = +u_z v_y + \xi u_z + \frac{-v_z u_z}{Tilt_1} + \frac{-u_z v_z}{Tilt_2} + \frac{-b_y}{Buoy} + \frac{F_i}{Fric},
\]

Fig. 6. Time–latitude plots of (a) SST, (b) low Rilow Ri layer thickness, (c) median Sh2 red within the low Ri layer (same color bar as in Fig. 7), (d) mean \( u_z \) in the top 60 m, (e) mean \( v_z \) in the top 60 m, and (f) mean \( N^2 \) within the low Ri layer for one TIW. Vertical lines mark time instants presented in Fig. 7. The black contours mark SSTs of 22.4° and 23.5°C.
Here, $\zeta = (f + u_x - u_z)$ is the absolute vorticity, $b_x$, $b_y$ are baroclinic torque terms, $F_x^i$, $F_y^i$ are the frictional terms, and $D_t = \partial_t + \partial_x + \partial_y + \partial_z$ is the material derivative.

We identify $u_x$, $v_y$, and $u_y$, $u_z$ as vortex stretching terms (STRETCH) since a component of horizontal flow divergence $v_y$ (or $u_x$) acts to stretch or squash a component of horizontal vorticity $u_x$ (or $v_x$), analogous to vertical divergence $w_z$ stretching or squashing vertical vorticity $v_x = -u_z$.

We identify $(\zeta - u_x)v_y$ and $(\zeta + u_x)u_y$ as tilting terms since they rotate one component of horizontal vorticity $v_y$ or $u_x$ to modify the other component $D_t u_x / Dt$ or $D_t v_y / Dt$. The tilting terms are decomposed into two (TILT1, TILT2). TILT1 $\approx 0$ because absolute vorticity $\zeta \approx 0$ in the cold cusp (Holmes et al. 2014). The TILT2 terms in Eqs. (6) and (7) do not simply rotate preexisting shear but also act to change the shear magnitude squared $S^2$ (horizontal enstrophy), which evolves according to

$$D_t S^2 = \frac{1}{2} \left( \frac{v_y^2 + v_x^2}{STRETCH} - \frac{(u_x + v_x)u_y}{TILT2} - \frac{u_x b_y - v_x b_y}{BUOY} + \frac{u_y F_x^i + v_y F_y^i}{FRIC} \right).$$

Note that TILT1 is absent in (8) even if absolute vorticity $\zeta \neq 0$. Taken together, the stretching and TILT2 terms in (8) act to increase $S^2$ throughout the deep cycle layer (not shown). Here the TILT2 terms appear as flow deformation in the $x$--$y$ plane $- (u_x + v_x)$ acting to stretch horizontal vortex lines in that plane ($u_x$, $v_x$), thereby changing the squared magnitude of horizontal vorticity and vertical shear $S^2$.

We present both components of vertical shear as well as the evolution terms in the RHS of Eqs. (6) and (7), averaged over the top 60 m, in Fig. 9 along with a contour marking a low Ri layer thickness of 30 m for reference. We focus on the magnitudes of the evolution terms within this 30-m low Ri layer.
thickness contour. In the northward-oriented cold cusp between 0° and 3°N, TIW meridional diffusivity (positive $v_z$) stretches the horizontal vorticity through the stretching term, $u_z$, $v_z$ (Fig. 9c), intensifying $u_z$ and forcing turbulence (Figs. 7e–h) as described by Holmes and Thomas (2015). North of approximately 2°N, the zonal shear weakens and meridional shear intensifies as the horizontal vorticity vector rotates from pointing southward to pointing westward. That is, negative $u_z$ is converted to positive $v_z$ in the northwest corner of the TIW (Fig. 8) through the horizontal vortex tilting terms for both $u_z$ and $v_z$ (TILT2, Figs. 9g,h). The sign of the TILT2 tilting terms within the 30 m contour is such that $u_z$ is increased from a negative value to 0 (its magnitude is decreased) while $v_z$ is increased from 0 to a positive value (Figs. 9a,b). Both tilting by absolute vertical vorticity (TILT1, Figs. 9e,f) and the baroclinic torque (BUOY, Figs. 9i,j) are relatively weak within the 30-m low Ri layer thickness contour. Enhanced turbulence within the 30-m low Ri layer thickness contour acts to decrease $S$^5 (Figs. 9k,l).

The intensification of $u_z$ in a near-equatorial region by horizontal vortex stretching (Holmes and Thomas 2015), the subsequent rotation of $u_z$ to $v_z$ by the anticyclonic TIW flow $u_z$ (Figs. 9g,h), and the intensification of shear through flow deformation is the continuous shear forcing that forces deep-cycle turbulence in the cold cusp.

c. Stratification in the cold cusp

Stratification $N^2$ in the low Ri layer decreases as water parcels flow northward and then eastward along the cold cusp (Figs. 6f and 10a). Consider the evolution equation for $N^2$, ignoring lateral mixing,

$$D_t N^2 = D_z b_z = -u_z b_z - v_z b_z + w_z b_z + \frac{\partial}{\partial t} K_{i} b_z. \quad (9)$$

Integrated over the low Ri layer, the stretching and tilting terms act to increase stratification (STRETCH + TILT, Fig. 10c) while the mixing acts to decrease stratification (MIX, Fig. 10d). In total, these terms act to decrease stratification within most of the low Ri layer (Fig. 10b). Consequently, a relatively weaker shear field can force turbulence in the cold cusp off the equator than on the equator, where stratification is stronger.

d. Simulated trigger of deep-cycle turbulence

As at the equator, turbulence in the low Ri layer must be triggered daily for a diurnal cycle to exist (Fig. 5). How does the KPP scheme model that daily trigger off the equator?

During times of TIW influence, our simulations show some diurnal variation in $H_{0.3}$, the shallowest depth at which $R_i$ = 0.3 (section 2a), in the TIW cold cusp off the equator (Fig. 11). However, the base of the KPP boundary layer $H_{KPP}$ is significantly shallower than $H_{0.3}$ during daytime (cf. black and thick red lines in Fig. 11). During daytime, the surface buoyancy forcing is stabilizing and values of $H_{KPP}$ are limited to the Monin–Obukhov scale $L_{MO}$ (section 2a), which is approximately 2.5–5 m (1–2 grid cells deep). After peak sun in the afternoon, $L_{MO}$ deepens to approximately 50–60 m and so does $H_{KPP}$ (Figs. 11b–e). After 1900 local time, the surface heat flux turns destabilizing, so $L_{MO}$ becomes negative, the limiter is removed and $H_{KPP}$ matches $H_{0.3}$ during the night. In this way, the descent of the daytime shear layer in the late afternoon (Smyth et al. 2013) is entirely modeled by the change in surface fluxes (as captured by $L_{MO}$) rather than a dynamical instability resulting from increased interior shear relative to the stratification (as captured by $H_{0.3}$).

Note that $H_{KPP} = L_{MO} > z_{MLD}$ for approximately an hour (1800–1900 local time) before the onset of convection (vertical white dashed lines). Diffusivities are enhanced within the
FIG. 9. Terms controlling the evolution of (left) $u_z$ [Eq. (6)] and (right) $v_z$ [Eq. (7)]. All quantities are depth-averaged in the top 60 m. Black contours mark low Ri layer thickness of 30 m for reference. Vertical lines mark the same time stamps as those in Fig. 7.
boundary layer, which includes the stratified water beneath the mixed layer, resulting in large heat fluxes just prior to the onset of nighttime convection. The descent of the boundary layer results in strong parameterized turbulence heat fluxes below the mixed layer where the strong vertical stratification combines with high boundary layer diffusivities (Fig. 11e). Parameterized mixing is also excited in the low Ri layer below $H_{KPP}$. During the night, $R_i$ increases which in turn reduces $J_t^q$ (Figs. 11b,e). Meanwhile $H_{KPP}$ shoals and approaches $z_{MLD}$ as it descends. When the sun rises, the $L_{MO}$ limiter is activated and the boundary layer shoals to its daytime value of approximately 5 m. Since $L_{MO} < H_0$, $H_{KPP}$ is set to $L_{MO}$ during the day. Daytime $R_i$ values in the low Ri layer are in the 0.4–0.5 range, so the parameterized shear turbulence is never completely extinguished ($KPP$ enhances diffusivities for $R_i < 0.7$). The daily cycle then repeats (Fig. 5).

There are two peaks in the parameterized turbulent heat flux $J_t^q$ averaged over the passage of the entire TIW associated with deep-cycle layers at the equator and in the cold cusp (Fig. 12a). Roughly one-third of the total off-equatorial parameterized heat flux in the low Ri layer between 3° and 5°N occurs within the KPP surface boundary layer (Fig. 12a). In contrast, this fraction is negligible near the equator. Equatorial deep-cycle turbulence is almost completely modeled by KPP’s interior shear mixing scheme. The difference between the modeled equatorial and off-equatorial deep-cycle turbulence is likely due to the difference in mean $N^2$, $S^2$ (Fig. 12b). Off the equator, $N^2$ is smaller than at the equator by approximately 70%, $S^2$ is smaller by approximately 40%, and $H_{0.3}$ penetrates deeper below the mixed layer compared to at the equator (Fig. 12c). So a larger fraction of the low Ri layer off the equator is handled by the surface boundary layer scheme. This difference in equatorial and off-equatorial $N^2$, $S^2$, $H_{0.3}$ also means that the $L_{MO}$ limiter is more consequential off the equator than at the equator, and may explain why Large and Gent (1999) did not see any sensitivity to the $L_{MO}$ limiter in their equatorial study (note that their calculations did not represent the influence of TIWs; their section 3d). Though the difference between $H_{KPP}$ and $z_{MLD}$ off the equator is small on average (cf. orange and black lines in Fig. 12c), the large diffusivities of the surface boundary layer scheme combine with small but nonzero stratification to yield large heat fluxes off of the equator (e.g., Fig. 11e).

Despite the bias in mean $R_i$ ($\sim 0.4$ versus the observed 0.25), the average $J_t^q$ at the equator (100–125 W m$^{-2}$, Fig. 12a) is within a factor of 2 of Warner and Moum’s (2019) observational estimate of average $J_t^q \sim 70–100$ W m$^{-2}$ right on the equator during La Niña conditions (when TIWs are active). It is possible that KPP is producing the right turbulent flux but for a flow with $R_i \sim 0.4$ instead of $R_i \sim 0.25$ by adjusting the diffusivity appropriately. Testing this idea would require a thorough comparison of a simulation covering multiple TIWs with the long-term microstructure observations presented in Warner and Moum (2019).

5. Indirect observational evidence

To our knowledge there is no reported observational evidence for an off-equatorial deep cycle of turbulence associated
with TIWs. In the absence of direct microstructure measurements, one might look at in situ Ri estimates since marginal stability is detectable using coarse measurements of shear and stratification at least at the equator (Smyth and Moum 2013; Pham et al. 2017). The absence of ADCPs on the off-equatorial TAO moorings prevents replicating the analysis of Smyth and Moum (2013) or Pham et al. (2017). Instead we will use coincident measurements of velocity from shipboard ADCPs and density from CTD casts obtained during cruises that sampled a TIW cold cusp by chance. Cruise CTD and ADCP data were obtained from the CLIVAR and Carbon Hydrographic Data Office (CCHDO) and the Joint Archive for Shipboard ADCP (JASADCP), respectively.

Ri estimated using high-resolution velocity and stratification measurements indicate that deep-cycle turbulence is associated with a Ri distribution with statistical mode 0.25 (2-m velocity bins; Smyth and Moum 2013). The mode remains at 0.25 even when the velocity measurements are significantly degraded to 16-m bins. The Smyth and Moum (2013) observations are limited in that they were taken at a single location (0°, 140°W) for a short period of time (2 weeks) during a period of strong TIW forcing. Guided by their observations, we assume that deep-cycle turbulence everywhere is characterized by a Ri distribution with statistical mode 0.25, and that Ri in the deep-cycle layer can be estimated from relatively coarse observations. Since the

Fig. 11. Turbulence at 3.5°N, 110°W triggered by deepening of the boundary layer in the afternoon. (a) Surface forcing with net surface heat flux in blue and wind stress in orange. (b) Gradient Ri, (c) $S^2$, (d) $N^2$, and (e) turbulent heat flux $J_t$. Panels (b)–(e) also show the turbulent heat flux $J_t$ (black contours; solid lines are negative, dashed are positive), the mixed layer depth ($z_{MLD}$, orange dashed line), the KPP boundary layer depth (black line), and the depth at which the bulk Ri reaches 0.3 ($H_{0.3}$, red line). Vertical white dashed lines mark time at which net surface heat flux changes sign.
mode is 0.25, we expect the most likely observed Ri value to be 0.25 in a deep-cycle layer.

We infer low values of Ri, less than 1, below the mixed layer both at the equator (as expected) as well as off the equator at 4° and 5°N in profiles from three cruise transects at 110°W through TIWs (Table 1; Fig. 13). We estimate Ri by first averaging the 1-m binned CTD data in the bins used for the ADCP data (either 8- or 10-m bins), and then estimating \( N^2 \) and \( S^2 \) using centered differences so the gradients are calculated over 16–20 m. Bins where \( N^2 \), \( 10^{-5} \) and \( S^2 \), \( 10^{-6} \) s\(^{-2} \) are excluded. These Ri profiles are presented in Figs. 13b,c,e,f,h,i along with a higher-resolution \( N^2 \) profile estimated using 3 m averaged CTD profiles. In general, Ri < 1 off the equator with values close to 0.25 in 3–5 successive bins below the mixed layer (where \( N^2 \approx 10^{-5} \) \( S^2 \), Figs. 13c,f,i). As in the model, \( \nu \) is the dominant shear term (not shown). The data in Fig. 13 are noisy but suggest that further examination of such sections might be useful given the lack of moored ADCPs and microstructure data. A more thorough analysis would use all sections at 110°, 125°, and 140°W to estimate a median Ri profile through the TIW cold cusp. This work is ongoing.

6. Summary

We have presented evidence from a high-resolution numerical simulation that TIWs force deep-cycle turbulence off

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<td>EP393</td>
<td>R/V Discoverer</td>
<td>24 Aug 1993–18 Sep 1993</td>
<td>WOCE PR16</td>
<td>8 m</td>
</tr>
<tr>
<td>RB0711</td>
<td>R/V Ron Brown</td>
<td>15 Dec 2007–18 Jan 2008</td>
<td>CLIVAR P18N</td>
<td>8 m</td>
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FIG. 13. Indirect evidence for an off-equatorial deep cycle from three cruise transects (Table 1). (a),(d),(g) SST from the daily 0.25° OISST product (Reynolds et al. 2007) with CTD stations marked (note changing color scale). (b),(c),(e),(f),(h),(i) Profiles of $R_i$ (black) and $N^2$ (blue) at stations marked with white circles in (a), (d), and (g). The vertical line is $R_i = 0.25$, and the horizontal line is mixed layer depth estimated as the shallowest depth at which the potential density exceeds the shallowest recorded density value by 0.015 kg m$^{-3}$. 
the equator throughout the cold tongue (Figs. 2–5, 11). Horizontal vortex stretching by meridional diffuence $v_z$ of the TIW’s flow intensifies preexisting $u_z$, within approximately $2^\circ$ of the equator (Fig. 9c; Holmes and Thomas 2015). The TIW’s anticyclonic flow then rotates the horizontal vorticity anticlockwise while moving fluid away from the equator, converting $u_z$ to $v_z$ in the eastward extension of the cold cusp (Figs. 8 and 9g–h). Flow deformation in the $x$–$y$ plane also acts to modify the horizontal vorticity in the cold cusp. This continuous stretching and tilting of horizontal vorticity acts to create a region of intense shear and low Ri below the mixed layer in the TIW cold cusp (Fig. 6). The flow regime in the TIW cold cusp is similar to the equatorial flow regime where intense shear ($u_z$) between the SEC and EUC exists below the mixed layer, except that the shear is now associated with meridional flow ($v_z$, Fig. 7). There is a daily cycle of turbulence in the off-equatorial low Ri layer that is associated with a downward descending shear layer in the afternoon (Fig. 5).

Although the simulated off-equatorial deep cycle has many qualitative similarities to the equatorial deep cycle, some details differ. Notably the descending surface boundary layer, which triggers deep-cycle turbulence, frequently descends with the Monin–Obukhov (MO) depth $L_{MO}$ as the stabilizing surface buoyancy flux weakens in the afternoon off of the equator. At the equator, in contrast, the descent of the stable surface boundary layer during the afternoon is usually (approximately two-thirds of the time) controlled by the bulk Richardson number, which is increasing momentum in the near-surface layer, or occurs with the onset of convection in the evening (as in Large and Gent 1999; Pham et al. 2017). Preliminary experiments without the $L_{MO}$ limiter indicate that the limiter can significantly influence the diurnal cycle of vertical mixing, consistent with the results of this paper (cf. $L_{MO}$ and $H_{0.5}$ in Fig. 11e). However, future work is required to clarify whether these differences persist once feedbacks develop, and how the $L_{MO}$ limiter impacts the model solution mean state, and the daily cycle of turbulence. In any case, the deepening boundary layer results in mixing in the low Ri layer beneath the MLD (Figs. 5 and 11; section 4d). Approximately one-third of the simulated off-equatorial turbulent heat flux is within the surface boundary layer, in contrast to the equatorial deep-cycle where the boundary layer accounts for a very small fraction of turbulent heat flux (Fig. 12a).

The accuracy of KPP’s simulation of these details of the off-equatorial deep cycle needs to be evaluated with LES simulations that account for the large-scale lateral TIW shear forcing. Most published LES simulations of deep-cycle turbulence do not account for lateral momentum and buoyancy flux forcing terms, and none that we know of account for TIW forcing explicitly. Observational evidence for an off-equatorial deep cycle is restricted to a small number of Ri profiles from opportunistic cruise transect data within a TIW cold cusp (Fig. 13). Similarly the horizontal vortex stretching and tilting dynamics described here and in Holmes and Thomas (2015) also need to observationally verified. Hence, there is a need for high-resolution microstructure observations to validate the current MITgcm and future LES simulations.

Deep-cycle turbulence transports heat absorbed by the near-surface ocean during daytime to depths deeper than the base of the convective mixed layer, keeping it away from reabsorption by the atmosphere during convection the following night. In doing so, deep-cycle turbulence helps keep the sea surface cool, enabling significant heat uptake by the eastern equatorial Pacific cold tongue while it remains in approximate thermal equilibrium. Our simulations indicate that this effective subduction of heat is taking place over an area larger than what might be expected from shear turbulence associated with SEC–EUC shear, i.e., poleward of the $3^\circ$–$3^\circ$N region (Figs. 2 and 3). Yet direct microstructure measurements have mostly been made at the equator. Our results, and the associated uncertainties, emphasize the need to constrain the magnitude, the dynamics, and the long term impact of off-equatorial turbulence in the eastern Pacific cold tongue.

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